

# Breaking parameter modulated chaotic secure communication system

G. Álvarez<sup>\*</sup>, F. Montoya, M. Romera, G. Pastor

*Instituto de Física Aplicada, Consejo Superior de Investigaciones Científicas,  
Serrano 144—28006 Madrid, Spain*

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## Abstract

This paper describes the security weakness of a recently proposed secure communication method based on parameter modulation of a chaotic system and adaptive observer-based synchronization scheme. We show that the security is compromised even without precise knowledge of the chaotic system used.

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## 1 Introduction

In recent years, a growing number of cryptosystems based on chaos have been proposed [1], many of them fundamentally flawed by a lack of robustness and security. In [2], the author proposes a symmetric secure communication system based on parameter modulation of a chaotic oscillator acting as a transmitter. The receiver is a chaotic system synchronized by means of an adaptive observer.

In this paper we show how to break the proposed cryptosystem when Lorenz's attractor is used as the non-linear time-varying system ([2, §3.2]), which, in fact, was the only example explained in detail. Lorenz system is described by the following equations:

$$\dot{x}_1 = -\sigma_1 x_1 + \sigma_2 x_2, \quad (1)$$

$$\dot{x}_2 = r x_1 - x_2 - x_1 x_3, \quad (2)$$

$$\dot{x}_3 = x_1 x_2 - b x_3. \quad (3)$$

In the example the system is implemented with the following parameter values,  $(\sigma_1, \sigma_2, r, b) = (10, 10, 28, 8/3)$ . The signal used for synchronization of the

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<sup>\*</sup> Corresponding author: Email: gonzalo@iec.csic.es

receiver is  $x_1$ . The encryption process is defined by modulating the parameter  $\sigma_1$  with the binary encoded plaintext, so that it is  $\sigma_1 + 2.5$  if the plaintext bit is "1" and  $\sigma_1 - 2.5$  if the plaintext bit is "0". The duration of the plaintext bits must be much larger than the convergence time of the adaption law. Actually, in the example the bit rate is 0.2 bits/second. The uncertain system can be rewritten in a compact form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\sigma_1 & \sigma_2 & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} (-y)\theta \quad (4)$$

$$y = C \cdot x = x_1 \quad (5)$$

$$C = [1 \ 0 \ 0] \quad (6)$$

$$\theta = \Delta\sigma_1 = \pm 2.5 \quad (7)$$

The decryption process consists of a chaotic system synchronized by means of an adaptive observer. The observer-based response system is designed as:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{bmatrix} = \begin{bmatrix} -\sigma_1 & \sigma_2 & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{pmatrix} 0 \\ -\hat{x}_1\hat{x}_3 \\ \hat{x}_1\hat{x}_2 \end{pmatrix} + LC(x_1 - \hat{x}_1) \quad (8)$$

$$L = [0 \ 38 \ 0]^T \quad (9)$$

The plaintext can be retrieved from the first derivative of the receiver uncertainty defined as:

$$\dot{\hat{\theta}} = -5y(x_1 - \hat{x}_1) \quad (10)$$

The initial conditions of the transmitter and receiver are:  $(x_1(0), x_2(0), x_3(0)) = (10, 15, 20)$  and  $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), \hat{\theta}(0)) = (0, 0, 0, 0)$ .

Although the author seemed to base the security of its cryptosystem on the chaotic behavior of the output of the Lorenz non-linear system, no analysis of security was included. It was not considered whether there should be a key in the proposed system, what it should consist of, what the available key space would be, and how it would be managed. We discuss the weaknesses of this secure communication system in Sec. 2 and in Sec. 3.

## 2 Power analysis attack

The main problem with this cryptosystem lies on the fact that the ciphertext is an analog signal, whose waveform depends on the system parameter values and therefore on the plaintext signal, which modulates one parameter. Consequently, the plaintext signal may be recovered from the transmitted signal power. Fig. 1 shows the Lorenz chaotic attractor for the different values of the parameter  $\sigma_1$  proposed by the author, making apparent the strong dependence of waveforms from the plaintext. In Fig. 1 (a) and (b) the attractor corresponding to  $\sigma_1 = 7.5$  and to  $\sigma_1 = 12.5$  are shown, respectively. We can observe that the signal amplitudes are quite different. In Fig. 1(c) the attractor trajectory corresponding to a modulation of the  $\sigma_1$  parameter between 7.5 and 12.5 is shown. We can observe that the resulting trajectory is the superposition of the two preceding trajectories, although both are clearly recognizable, allowing the easy separation of each other.

To break the system we have implemented the chaotic transmitter of the author's example with the same parameters values and initial conditions. The simulation is identical to the one employed in the original example, a four order Runge-Kutta integration algorithm in MATLAB 6. A step size of 0.001 was used.

To recover the plaintext we used no chaotic receiver, instead we computed the short time power analysis of the ciphertext. The procedure is illustrated in Fig. 2. The first step consists of squaring the ciphertext signal,  $x_1$ . Next, this signal is low-pass filtered and, finally, binary quantized. The low-pass filter employed is a four pole Butterworth with a frequency cutoff of 0.5 Hz. The quantizer is an inverting Smith-trigger with switch on point at 80 and switch off point at 50.

The result is a good estimation of the plaintext, with tiny inaccuracies consisting of small delays in some transitions. Note that the short initial error was also present at the beginning of the retrieved signal obtained with the authorized receiver described in the author's example.

It should be emphasized that our analysis is a blind detection, made without the least knowledge of what kind of non-linear time-varying system was used for encryption, nor its parameters values, and neither its keys, if any.

### 3 Generalized Synchronization attack

A more precise signal retrieving of the plaintext can be performed if we know what kind of non-linear time-varying system was used for encryption, but still without the knowledge of its parameter and initial condition values.

We have implemented another attack by means of an intruder receiver based on generalized synchronization [3], fairly simpler than the authorized receiver. We use the following receiver:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{bmatrix} = \begin{bmatrix} -\sigma_1 & \sigma_2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{pmatrix} 0 \\ rx_1 - x_1\hat{x}_3 \\ x_1\hat{x}_2 \end{pmatrix} \quad (11)$$

The plaintext recovery procedure consists of the estimation of the short time cross correlation between the ciphertext and the recovery error. It is illustrated in Fig. 3. The first step consists of calculating the synchronization error of the receiver  $\Delta x_1 = x_1 - \hat{x}_1$ . Next the synchronization error  $\Delta x_1$  is multiplied by the ciphertext  $x_1$ . Then this signal is low-pass filtered. Finally, a binary quantizer is used to regenerate the plaintext. The low-pass filter employed is a four pole Butterworth with a frequency cutoff of 0.5 Hz. The binary quantizer is a Smith-trigger with switch on point at 11 and switch off point at 9.

We have found that the sensitivity to the parameter values is so low that the original plaintext can be recovered from the ciphertext using a receiver system with parameter values considerably different from the ones used by the sender. The parameter values can be obtained with a very accurate precision by means of the trial and error procedure varying them in an effort to approximate the filter output signal to a square wave. However, their exact knowledge is not necessary to recover the plaintext, as already illustrated in Fig. 3.

The approximate range of parameter values that causes a chaotic behavior of the Lorenz oscillator is:

$$\sigma_1 = \{4, 14\}, \quad (12)$$

$$\sigma_2 = \{8, 30\}, \quad (13)$$

$$r = \{24, 90\}, \quad (14)$$

$$b = \{1.5, 4.5\}. \quad (15)$$

Actually, we have selected for our implementation, represented in Fig. 3, the central value of each of the preceding parameter ranges, that is:  $(\sigma_1, \sigma_2, r, b) = (9, 19, 66, 3)$ , with initial conditions  $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), \hat{\theta}(0)) = (0, 0, 0, 0)$ .

## 4 Conclusions

The proposed cryptosystem is rather weak, since it can be broken without knowing its parameter values and even without knowing the transmitter precise structure. There is no mention about what the key is, nor which is the key space, a fundamental aspect in every secure communication system. The total lack of security discourages the use of this algorithm for secure applications.

## Acknowledgements

This work is supported by Ministerio de Ciencia y Tecnología of Spain, research grant TIC2001-0586.

## References

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- [2] Moez Feki, An adaptive chaos synchronization scheme applied to secure communication. *Chaos, Solitons and Fractals* **18** (2003) 141–148.
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## Figure captions

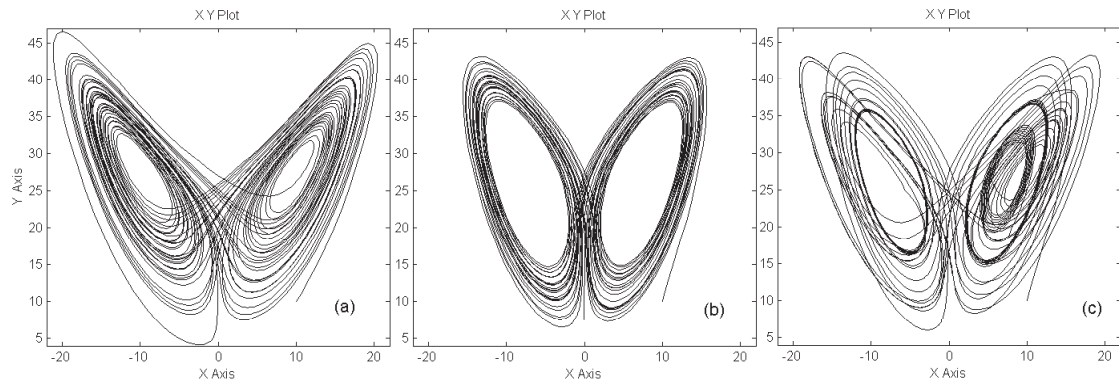


Fig. 1. Lorenz attractor with different parameter values: (a)  $\sigma_1 = 7.5$ ; (b)  $\sigma_1 = 12.5$ , (c)  $\sigma_1$  is switched between 7.5 and 12.5 by the plaintext.

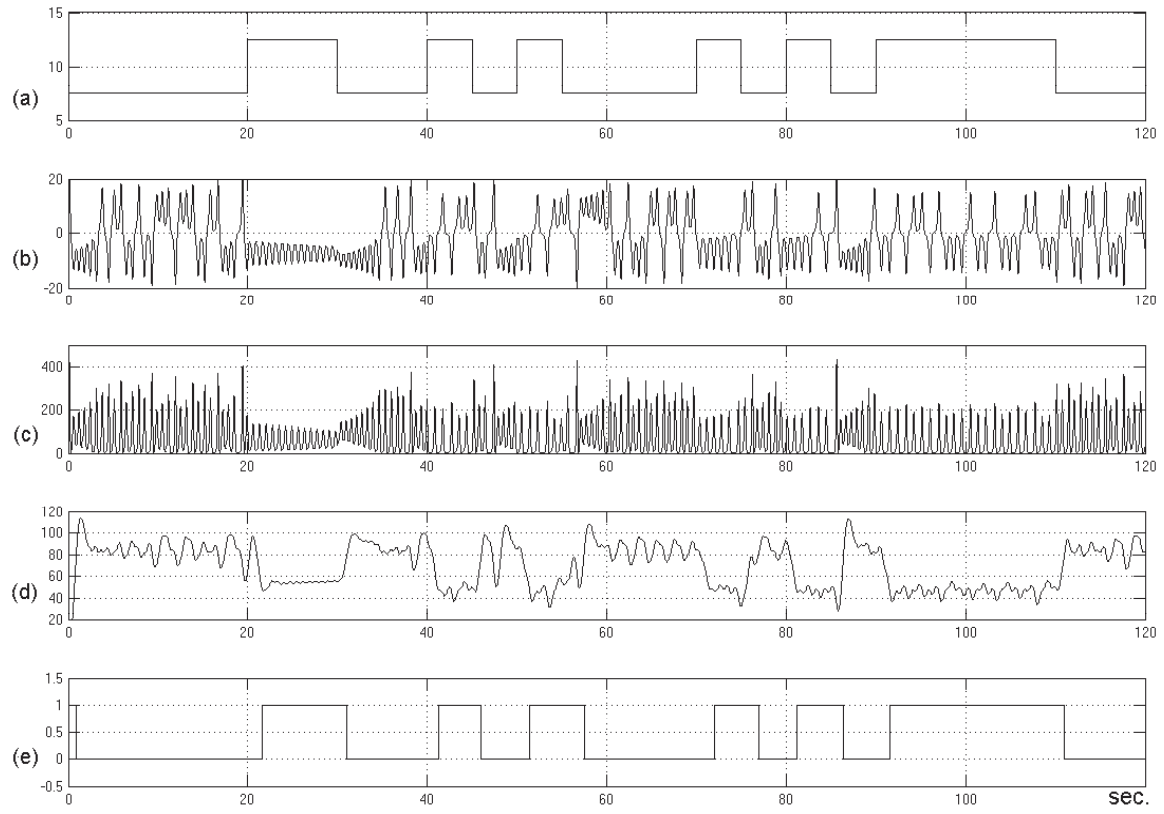


Fig. 2. Power signal attack: (a) plaintext; (b) ciphertext,  $x_1$ ; (c) squared ciphertext signal,  $x_1^2$ ; (d) low pass filtered squared ciphertext signal; (e) recovered plaintext.

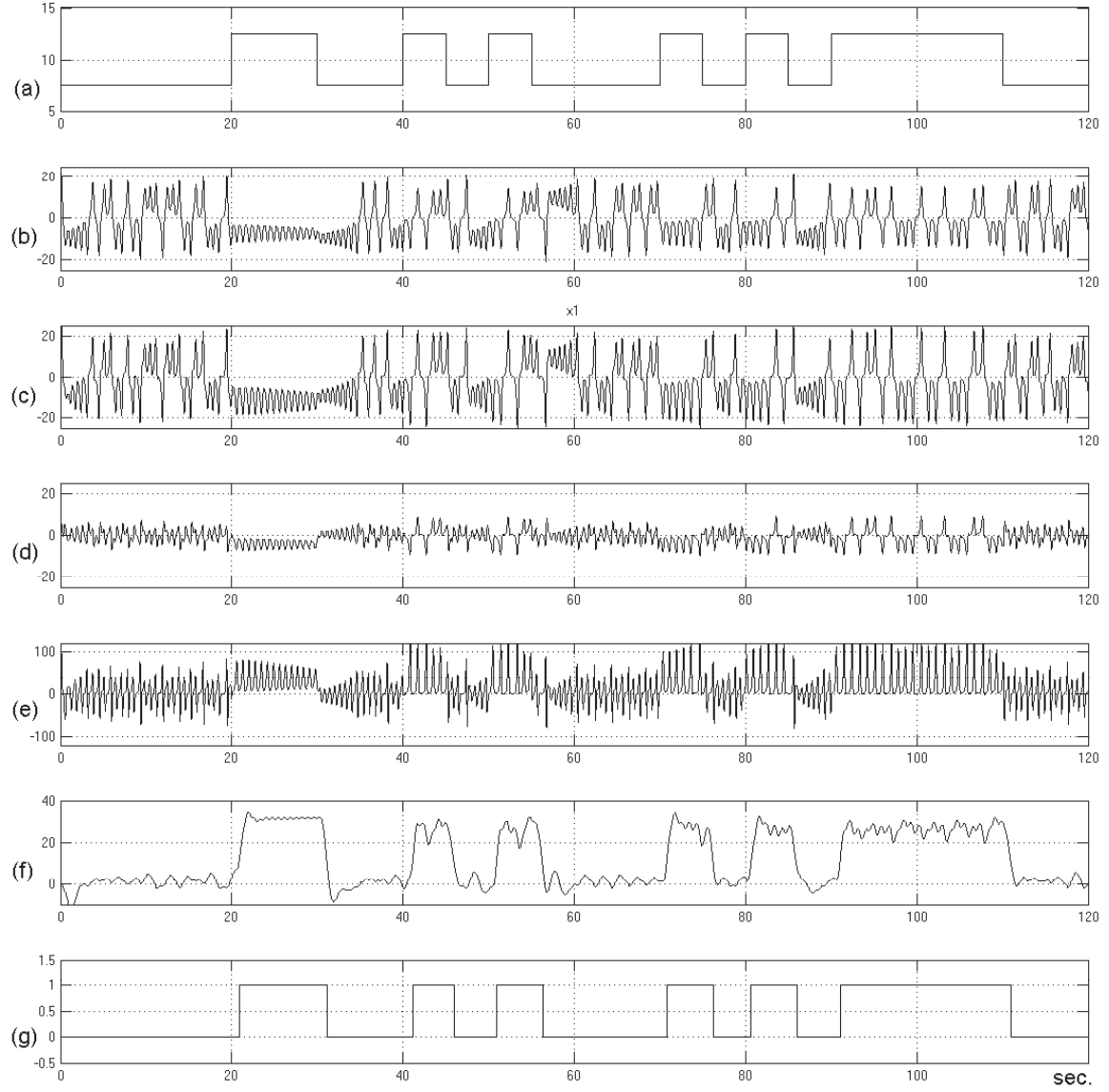


Fig. 3. Generalized Synchronization attack: (a) plaintext; (b) ciphertext,  $x_1$ ; (c) signal generated by the intruder's receiver,  $\hat{x}_1$ ; (d) synchronization error of the intruder's receiver,  $\Delta x_1 = x_1 - \hat{x}_1$ ; (e) ciphertext multiplied by synchronization error,  $x_1 \cdot \Delta x_1$ ; (f) low-pass filtering of (e); (g) recovered plaintext.